Black Hole in Higher Curvature Gravity and AdS/CFT Correspondence

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The classical central charge for the higher curvature gravity in 3-dimensions is calculated using the Legendre transformation method. The statistical entropy of BTZ black hole is derived by the Cardy's formula and the result completely coincides with the Iyer-Wald formula for the geometrical entropy. This coincidence suggests the generalized AdS/CFT correspondence.

1 Introduction

It is now well established that there exists the correspondence between the Supergravity theory on Anti-deSitter spacetime (AdS) and the Large N limit of Super Yang-Mills CFT.[1] In the case of 3-d BTZ black hole , Strominger has given a simple statistical derivation of the Bekenstein-Hawking entropy via Cardy's formula.[2] This is an important check for the correspondence.

The AdS/CFT correspondence conjectured by Maldacena is the correspondence between the Type IIB Superstring theory on AdS and the Super Yang-Mills CFT. The string theory in general receives corrections like as $S_{\text{low energy}} = R + \alpha' R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \cdots$. Hence, a modest first step to the final aim is to extend AdS/CFT correspondence to higher curvature gravity. The geometrical entropy in the higher curvature gravity can be calculated by using the Noether charge method.[3] The purpose of this paper is to give an evidence of AdS/CFT correspondence in higher curvature gravity by deriving the Iyer-Wald formula in 3-dimensions using AdS/CFT correspondence.[4]

2 BTZ Black Hole Entropy in the Higher Curvature Gravity

Let us calculate the statistical entropy of the BTZ black hole in the higher curvature gravity. The action we treat here is of the form: $I = 1/16\pi G \int d^3x \, f(R, R_{\mu\nu}) \sqrt{-g}$ where $f(R, R_{\mu\nu})$ is the Lagrangian of general higher curvature gravity. This action is the most generic form of the three dimensional higher curvature gravity because the Weyl tensor vanishes in three dimensional spacetime. We restrict our treatment hereafter to the case that the spacetime is of the constant curvature. Further we assume that the curvature R is of negative. Under such conditions, the BTZ black hole can exist. The metric of the BTZ Black Hole is given by

$$ds^2 = -((\frac{r}{l})^2 + (\frac{4GJ}{r})^2 - \frac{GM}{l})dt^2 + \frac{dr^2}{(\frac{r}{l})^2 + (\frac{4GJ}{r})^2 - \frac{GM}{l}} + r^2 \left[d\phi - \frac{4GJ}{r^2}dt \right]^2 \quad (1)$$

where l is the curvature scale of the Anti-deSitter spacetime. Here mass M and the angular momentum J characterize the black hole. The event horizon can be read off from the metric as $r_+ = \sqrt{2Gl(M+J)} + \sqrt{2Gl(M-J)}$. The geometrical entropy of the black hole can be calculated to be

$$S_{IW} = -\frac{1}{8G} \oint_H d\phi \sqrt{g_{\phi\phi}} \frac{\partial f}{\partial R_{\mu\alpha}} g^{\nu\beta} \epsilon_{\mu\nu}\epsilon_{\alpha\beta} = \frac{1}{12G} g^{\mu\nu} \frac{\partial f}{\partial R_{\mu\nu}} \oint_{r_+} d\phi \sqrt{g_{\phi\phi}} \quad (2)$$

where H, h and $\epsilon_{\mu\nu}$ are the spatial section of the event horizon, the determinant of the induced metric on H, and the binormal to H, respectively.

In order to calculate the central charge for this theory, Legendre transformation[5] $\bar{g}^{\mu\nu} \equiv [-\det(\partial(f\sqrt{-g})/\partial R_{\alpha\beta})]^{-1} \partial(f\sqrt{-g})/\partial R_{\mu\nu}$ is useful. The point is that the action can be rewritten as the Einstein-Hilbert action of $\bar{g}^{\mu\nu}$ and an auxiliary matter field. Because the spacetime of the BTZ black hole is of constant curvature, substituting $\bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ into the Legendre transformation formula gives $\Omega = g^{\mu\nu}\partial f/\partial R_{\mu\nu}/3$. The calculation of the central charge in the Einstein frame can be carried out by relating the quantities in the Einstein frame to those in the original frame through the conformal transformation. The central charge is calculated to be $c = l/2G\, g^{\mu\nu}\partial f/\partial R_{\mu\nu}$ Although this is the central charge calculated in the Einstein frame, this central charge is equivalent to that in the original higher curvature frame.

The mass and the angular momentum in the higher curvature gravity are calculated through the Noether charge form to be ΩM and ΩJ, respectively. Then the two Virasoro eigen values λ and $\tilde{\lambda}$ in the higher curvature gravity are given by, $\lambda = \Omega(M+J)/2$, $\tilde{\lambda} = \Omega(M-J)/2$. Finally the statistical entropy of the BTZ black hole in the higher curvature gravity can be deduced from Cardy's formula[6]

$$S_C = 2\pi\sqrt{\frac{c\lambda}{6}} + 2\pi\sqrt{\frac{c\tilde{\lambda}}{6}} = \frac{\pi}{12G} g^{\mu\nu} \frac{\partial f}{\partial R_{\mu\nu}} \left[\sqrt{8Gl(M+J)} + \sqrt{8Gl(M-J)} \right] \quad (3)$$

This formula coincides with the statistical entropy S_{IW}.[4]

3   Summary

We have shown $S_{\text{IW}} = S_{\text{statistical}}$ in 3-dimensional higher curvature gravity. **This is an evidence of AdS/CFT correspondence in higher curvature gravity.** This also suggests the string-CFT correspondence.

References

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